M.Sc. (Mathematics) (NEP Pattern) Semester-III

03NEPMATH02 - Partial Differential Equations

P. Pages : 2 GUG/S/25/16014

Time : Three Hours * 3 6 2 6 * Max. Marks : 80

Notes :
1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that singular integral is also a solution of the first order partial differential equation. **8**

b) Find the integral of the Pfaffian differential equation $yzdx + xzdy + xydz = 0$. **8**

OR

c) Show that : A necessary and sufficient condition for the two partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible is that **8**

$$[f, g] = \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$$

d) Find the complete integral of the partial differential equation $z\left(p^2 + q^2\right) + px + qy = 0$. **8**

UNIT – II

2. a) Find the integral surface passing through the circle $z = 1, x^2 + y^2 = 1$ of the partial differential equation $(x - y)p + (y - x - z)q = z$. **8**

b) Solve the equations $z_x + z_y = z^2$ with the initial condition $z(x, 0) = f(x)$. **8**

OR

c) Find by the method of characteristics, the integral surface of pq = xy which passes through the curve z = x, y = 0. **8**

d) Prove that : The characteristic curves $x = X(s, t), y = Y(s, t), z = Z(s, t)$ from an integral surface of a non-linear partial differential equation. If s and t are solved in terms of x and y from the first two equations, then the last equation gives the solution expressed in the form $z = Z(s(x, y), t(x, y)) = z(x, y)$. **8**

UNIT – III

3. a) Derive the second order partial differential equation which describes the temperature distribution in a homogeneous isotropic solid. **8**

b) Reduce the equation $y^2u_{xx} - 2xy\,u_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ into canonical form and hence solve it. **8**

OR

c) Obtain D'Alembert's solution of the one dimensional wave equation which describes the vibrations of a semi-infinite string. **8**

d) Solve $y_{tt} - c^2y_{xx} = 0, \quad 0 < x < 1, \quad t > 0,$ **8**

$$y(0,t) = y(1,t) = 0,$$

$$y(x,0) = 0, \qquad 0 \le x \le 1,$$

$$y_t(x,0) = x^2, \qquad 0 \le x \le 1,$$

UNIT – IV

4. a) Show that the solution of the Neumann problem is unique up to the addition of a constant. **8**

b) Find the solution of the problem $\nabla^2 u = 0, \ -\infty < x < \infty, y > 0$, $u(x,0) = f(x), -\infty < x < \infty$, such that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$. **8**

OR

c) State and prove the Harnack's theorem. **8**

d) Find the temperature distribution in a rod of infinite length satisfying the initial conditions $u(x,0) = f(x), -\infty < x < \infty$. **8**

5. a) Eliminate the parameters a and b from the equation $z = (x+a)(y+b)$ and find the corresponding partial differential equation. **4**

b) Define : **4**
i) Semi-linear partial differential equation
ii) Quasi-linear partial differential equation.

c) Show that in the analytic function of $z = x + iy, f(z) = u(x,y) + iv(x,y)$, u and v satisfy two-dimensional Laplace's equation. **4**

d) Show the necessary condition for the existence of the solution of the Neumann problem is that the integral of f over the boundary B should vanish. **4**

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